Dimensionality Assessment: Additional Methods

In Chapter 3 we use a nonlinear factor analytic model for assessing dimensionality. In this appendix two additional approaches are presented. The first strategy is a traditional linear factor analytic approach and the second is structural equation model (SEM). As mentioned in Chapter 3 the linear factor analysis of dichotomous data runs the risk of identifying "difficulty" factors. However, we present the traditional approach because of its familiarity, the fact that its use with dichotomous data is not always problematic, and its generalizability to ordered polytomous data. That is, with ordered polytomous data linear factor analysis as well as SEM are completely appropriate. We conclude this appendix with a brief discussion of essential unidimensionality.

Dimensionality Assessment: Linear Factor Analysis

Recall that the mathematics data consist of five items administered to 19,601 examinees. The intercorrelations amongst the items and the descriptive statistics are presented in Table C.1; the top half of the table contains the instrument's correlation matrix, **R**. Although there are various ways one might assess the dimensional structure of an instrument's data (cf. Hattie, 1985; Stout, 1987, 1990; van den Wollenberg, 1988), one approach is to perform a principal axis (PA) analysis of the instrument's correlation matrix; see McDonald (1981) for a commentary on this approach.[1]

Our PA of **R** shows a Kaiser-Meyer-Olkin measure of sampling adequacy of 0.72734. This value indicates borderline acceptance of our sample for performing a PA. The first factor extracted has an eigenvalue (λ_1) of 1.2845 and accounts for 25.7% of the common variance. The remaining factors have λ s substantially less than 1.0. Lord (1980) suggests that if the first eigenvalue is large compared to the second and the second eigenvalue is not much larger than any of the others, then the instrument may be considered approximately unidimensional. The Cattell scree plot (Figure C.1) shows this pattern. Although λ_1 is not very large, the scree plot indicates that a single factor underlies the data. (Strictly speaking, Lord's guideline involves the eigenvalues from a tetrachoric item intercorrelation matrix; a tetrachoric correlation is defined in Appendix E: *Using Principal Axis for Estimating Item Discrimination*.)

Table C.1. Descriptive statistics and correlation matrix for the mathematics test data

	Items Intercorrelations				
	1	2	3	4	5
	1.0000				
	0.2329	1.0000			
	0.1816	0.3278	1.0000		
	0.1436	0.3263	0.3087	1.0000	
	0.1088	0.2273	0.2324	0.2372	1.0000
Mean (*P*)	0.8875	0.6441	0.5660	0.4270	0.3873
SD[a]	0.3160	0.4788	0.4956	0.4947	0.4872
r_c[b]	0.2460	0.4390	0.4157	0.4051	0.3117
biserial r	0.4070	0.5640	0.5240	0.5110	0.3970
communality	0.1031	0.3641	0.3140	0.2946	0.1607
loadings[c]	0.3211	0.6034	0.5603	0.5427	0.4008

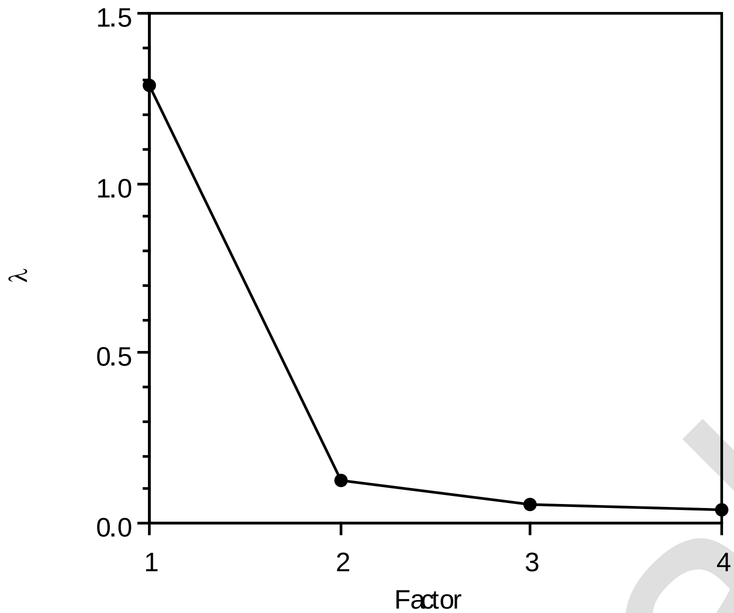
[a]Standard deviation

[b]Corrected item-total correlation (r_c; see Henrysson, 1963)

[c]Factor loadings on first factor

Based on this preliminary analysis we perform a second PA and specify the retention of only one factor. The reproduced **R** from the one factor solution shows that all residuals (the difference between **R** and the reproduced **R**) are less than |0.007|; the λ_1 is 1.2364 and accounts for 24.7% of the common variance. The loadings from the one factor solution are presented in Table C.1. Using a criterion that item loadings greater than 0.50 are considered good, then items 2-4 have good loadings, whereas items 1 and 5 have moderate loadings (i.e., using a moderate loading criterion of 0.30). Therefore, although the first factor has a low eigenvalue, the change between it and the remaining factors' eigenvalues is comparatively large and the one-factor solution is able to reproduce **R** relatively well. As a result, it appears that a unidimensional model is a sufficiently accurate representation of the data. The instrument's coefficient alpha is 0.6077 indicating a moderate degree of internal consistency given the short length of the examination.

Figure C.1. Scree plot for PA of Mathematics test data.



Dimensionality Assessment: Structural Equation Modeling

So far we have used linear and nonlinear factor analyses for assessing dimensionality. For pedagogical reasons we present an additional strategy using structural equation modeling . Admittedly, with only five items this is not the best SEM example. For our analysis we examine one factor and multiple two factor models. The two factor models are made up of every possible combination of two- and three-item factors (i.e., pass 1: factor I consists of items 1 and 2, factor II is identified by items 3-5; pass 2: factor I is defined in terms of items 1 and 3, factor II consists of items 2, 4, and 5, etc.). This latter (exploratory) approach is adopted because there is no theory or rationale to indicate which items should load on which factor. (If one had theory to guide the model development, then the process would be modified to incorporate this information.)

Each model produces a set of fit indices that are compared across the one- and two-factor models to obtain evidence supporting or disconfirming a unidimensional latent space. To perform the analyses MPLUS is used. The results are presented in Table C.2. The values of the fit measures (RMSEA, SRMR and TLI and given current guidelines for "good" fit: RMSEA < 0.06, SRMR < 0.09, and TLI > 0.96) indicate that both the one- and two factor models are acceptable. These results seem to indicate that the unifactor model fits the data reasonably well. In addition, the factor intercorrelations for the two-factor model are, in general, greater than

0.929. This also points toward a one factor model. Therefore, consistent with the previous dimensionality analyses it appears appropriate to proceed with our IRT calibration. In addition, a validity study of the calibration $\hat{\theta}$s would provide useful information as part of the dimensionality analysis. It should be noted that if the analysis had indicated that a particular two-factor model fit better (significantly or meaningfully) than the other models, then one would have some evidence indicating that there are two latent variables underlying our data. A validity study should also support this interpretation.

Table C.2. SEM One- and Two-Factor Model Results.

Normal Theory ML and Satorra-Bentler Robust Statistics

One Factor Model

X^2	CFI	TLI	RMSEA	SRMR	Scaling Factor	Normal Theory ML X^2
111.414	0.988	0.977	0.033	0.014	1.06	118.063

Two Factor Model

I Items	II Items	X^2	CFI	TLI	RMSEA	SRMR	$r_{I,II}$	Scaling Factor	
1, 2	3, 4, 5	107.411	0.989	0.972	0.036	0.013	0.967	1.061	
1, 3	2, 4, 5	109.543	0.988	0.971	0.037	0.013	0.976	1.053	
1, 4	2, 3, 5	113.458	0.988	0.970	0.037	0.014	0.999	1.041	
1, 5	2, 3, 4	101.108	0.989	0.973	0.035	0.013	1.000	1.044	*
2, 3	1, 4, 5	92.661	0.990	0.976	0.034	0.012	0.929	1.056	
2, 4	1, 3, 5	94.530	0.990	0.975	0.034	0.013	1.000	1.089	*
2, 5	1, 3, 4	95.853	0.990	0.975	0.034	0.013	1.000	1.068	*
3, 4	1, 2, 5	60.584	0.994	0.984	0.027	0.010	1.000	1.063	*
3, 5	1, 2, 4	112.237	0.988	0.970	0.037	0.014	1.000	1.051	*
4, 5	1, 2, 3	29.015	0.997	0.993	0.018	0.007	0.842	1.037	

Chi-Square Difference Tests

I Items	II Items	Normal Theory ML X^2	Difference Test Scaling Factor	X^2 Difference	Difference Test p
1, 2	3, 4, 5	113.979	1.056	3.867	0.049
1, 3	2, 4, 5	115.329	1.088	2.513	0.113
1, 4	2, 3, 5	118.059	1.136	0.004	0.953
1, 5	2, 3, 4	105.575	1.124	11.110	0.001
2, 3	1, 4, 5	97.854	1.076	18.782	0.000
2, 4	1, 3, 5	102.921	0.944	16.040	0.000
2, 5	1, 3, 4	102.37	1.028	15.266	0.000
3, 4	1, 2, 5	64.401	1.048	51.204	0.000
3, 5	1, 2, 4	117.974	1.096	0.081	0.776
4, 5	1, 2, 3	30.089	1.152	76.366	0.000

*Factor correlation exceeded 1.0
$r_{I,II}$: Factor intercorrelation

Robust Weighted Least Squares

One Factor Model

	X^2	CFI	TLI	RMSEA	SRMR
	61.339	0.995	0.992	0.024	0.022

Two Factor Model

I Items	II Items	X^2	CFI	TLI	RMSEA	SRMR	$r_{I,II}$	
1, 2	3, 4, 5	55.280	0.996	0.991	0.026	0.022	0.963	
1, 3	2, 4, 5	58.549	0.995	0.991	0.026	0.022	0.981	
1, 4	2, 3, 5	62.445	0.995	0.990	0.027	0.022	0.993	
1, 5	2, 3, 4	37.398	0.996	0.993	0.024	0.019	1.000	*
2, 3	1, 4, 5	54.884	0.996	0.991	0.025	0.021	0.962	
2, 4	1, 3, 5	53.584	0.996	0.992	0.025	0.021	1.000	*
2, 5	1, 3, 4	52.999	0.996	0.992	0.025	0.021	1.000	*
3, 4	1, 2, 5	34.108	0.997	0.995	0.020	0.017	1.000	*
3, 5	1, 2, 4	60.225	0.995	0.991	0.027	0.022	0.993	
4, 5	1, 2, 3	17.408	0.999	0.998	0.013	0.012	0.885	

*Factor correlation exceeded 1.0
$r_{I,II}$: Factor intercorrelation

Summary

We have analyzed our mathematics data using three different approaches. All of our approaches have converged to provide evidence supporting the use of a unidimensional model. For those situations where one has evidence that supports a multidimensional latent space there are a couple of strategies that one might consider depending on the form of multidimensionality. For instance, when items are associated with different latent variables one could perform separate calibrations for each latent variable utilizing only the items defining a particular latent variable. In short, one decomposes the instrument into unidimensional components and applies a unidimensional IRT model to each component. In this case, each person would have a profile of $\hat{\theta}$s describing his or her locations on the different latent variables. Whether it is meaningful to form a composite of these $\hat{\theta}$s so that each person has a single value is context specific as well as validity issue. As has been mentioned in the text, to the extent that the various dimensions are not orthogonal, then this approach of creating a set of unidimensional scales may not completely capture all the available information about the individuals. A second form of multidimensionality is, in effect, the opposite of the first. That is, in this case it is not possible to decompose the instrument into unidimensional components because the latent variables' interaction is manifested in the observed data. In this case, the use of a multidimensional IRT (Chapter 10) may be fruitful.

Endnotes

[1]The correlation matrix is obtained by calculating all pairwise Pearson correlations. Because the data are dichotomous this results in a matrix of phi correlation coefficients. A phi correlation coefficient is the application of the Pearson product-moment correlation to data that are true dichotomies. A variable is said to be a true dichotomy when it has only two *possible* values. The variables gender and cell phone ownership would be examples of true dichotomies. As such, the relationship between gender (i.e., male/female) and cell phone ownership (i.e., Yes/No) may be assessed using a phi coefficient. In addition, the linear factor analysis of a tetrachoric correlation matrix instead of a phi correlation matrix has also been recommended for dimensionality assessment; the tetrachoric correlation coefficient is discussed in Appendix E: *Using Principal Axis for Estimating Item Discrimination*. However, as is the case with the factor analysis of phi coefficients, it is still possible to observe difficulty factors with the factor analysis of a tetrachoric correlation matrix (Gourlay, 1951).

References

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